GPS-Based Attitude Determination for a Spinning Rocket

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BIOGRAPHIES

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ABSTRACT

An algorithm is developed for determining the attitude of a spinning sounding rocket. This algorithm is able to track signals with low availability duty cycles, but with enough accuracy to yield phase observables for the precise, 3-axis attitude determination of a nutating rocket. Raw GPS RF front-end data are processed by several filters to accomplish this task. First, a Levenberg-Marquardt algorithm (LMA) batch filter performs a least-squares fit on a bank of correlators to generate GPS observables for multiple satellites. These observables are then used as measurements in a Rauch-Tung-Striebel (RTS) smoother that optimizes estimates of carrier phase, Doppler shift, and code phase. Finally, attitude determination is carried out by another batch filter that uses the single-differenced optimization carrier phase estimates between two antennas. The batch filter itself is a combination of a substantially modified form of the LMA and the Least-Squares Ambiguity Decorrelation (LAMBDA) method. It is configured to deal with integer ambiguities that are constantly changing due to the long data gaps between times of carrier phase availability. The algorithm presented in this paper is applied to recorded RF data from a spinning sounding rocket mission in order to produce attitude quaternion and spin-rate estimates. These results are confirmed by another set of quaternions and spin-rate vectors independently estimated from magnetometer and horizon crossing indicator data.

I. INTRODUCTION

The work of the present paper pertains to the task of post-flight attitude determination for a sub-orbital sounding rocket mission: The Magnetosphere-Ionosphere Coupling in the Alfvén resonator (MICA) mission. MICA was launched from the Poker Flat Research Range in Fairbanks, Alaska on Feb. 19,
The rocket was spin-stabilized about its minor inertia axis, and consisted of a main payload and a sub-payload. The sub-payload carries two GPS antennas linked to two RF front-ends with a common clock, a magnetometer, and a horizon crossing indicator, as shown in Fig. 1.

![Rocket layout](image)

**Fig. 1: Rocket layout.**

The sub-payload ejects from the rest of the rocket at \( t \approx 98 \) s, and its E-field wire booms are fully extended by \( t \approx 168 \) s. This sub-payload is the vehicle that, for any given epoch time \( t_0 \), the attitude quaternion \( q_0 \) and spin-rate vector \( \omega_0 \) must be estimated. These quantities at any time \( t \) can then be found by propagation of the dynamic model discussed in Section III. However, there are a few major challenges that need to be overcome before this can be achieved for the MICA mission. First, the rocket undergoes substantial coning motion during its flight. Energy dissipation in the flexible booms leads to growing nutation, a characteristic of minor-axis spinners. Therefore, it cannot be assumed that rotation is restricted to the nominal spin axis. This presents a difficulty because the on-board GPS receiver is connected to only two antennas, so no single epoch can fully determine the required 3-axis attitude and spin-rate. The fact that the rocket is spinning can be exploited to estimate full 3-axis attitude and spin rate from only two antennas by using an Euler rotation dynamics model and differential carrier phase, but this leads to further complications.

The second challenge lies in the inherent nature of using differential carrier phase measurements: Resolving integer ambiguities. The considerable number of integer ambiguities in each batch of measurements change every time the signal data stream is interrupted. This is an issue in a low duty cycle scenario, as these integers must be re-estimated after every data gap, which leads to the third and most problematic challenge.

Telemetry bandwidth constraints within the sub-payload system reduces the received signal data’s availability to brief, periodic segments of about 0.0228 s of data every 0.5 s. This translates to a duty cycle of about 5%. Data gaps in the highly dynamic signal retrieved by a rapidly moving receiver require constant signal to initiate a re-acquisition. A standard procedure of re-acquisition phase-locked tracking loop in this situation might need a large amount of processing to acquire with a very fine correlator grid. Conversely, a PLL might need a large amount of time to settle after a coarse acquisition. The former approach is inefficient and would be impractical for a real-time system, while the latter would not yield useful data from the rocket’s short bursts of GPS data availability.

The contribution of this paper is the ability to handle the data gaps in the presence of the other challenges. To the best of the authors’ knowledge, the present work will be the first to demonstrate two-antenna attitude determination using actual flight data that is not continuous. Another work has investigated the use of differential carrier phase in a two-antenna system for attitude determination, but it needed continuous data from phase-locked loops for long periods of time [1]. It did not have to deal with data gaps and frequent changes of integer ambiguities.

A strategy is designed in which a coarse acquisition is followed by calculations that involve a series of correlator banks, the theory behind which is given in detail in Ref. [2]. These banks contain enough offsets in Doppler shift and code phase to cover the uncertainty in the location of the signal’s correlation peak. GPS observables at this peak are found by an LMA batch filter and further optimized by an RTS smoother. One of the observables is carrier phase, which is single-differenced between the two antennas to eliminate common receiver errors and to provide subcentimetre accuracy. This is done for four or more satellites to ensure the problem is observable. The raw phase difference measurements between the two antennas for each satellite are compared to a differential carrier phase model in a final LMA nonlinear least-squares batch filter that simultaneously estimates \( q_0, \omega_0 \), and the integer ambiguities.
The remainder of the paper is divided into four sections. The next section examines how the carrier phase measurements are extracted through the LMA batch filter and RTS smoother. Section III defines the carrier phase model used by the final batch filter to obtain attitude and spin-rate results. Section IV describes the attitude batch filter itself, and Section V presents the results from applying the entire algorithm to the data recorded on MICA.

II. GPS SIGNAL DATA PROCESSING

Levenberg-Marquardt Batch Optimization

As previously mentioned, GPS signal data is only available in bursts of approximately 0.0228 s segments at 0.5 s intervals. To manage these data interruptions, the algorithm begins by performing an FFT acquisition at the beginning of each segment. The Doppler shift output here needs only to be a rough estimate, as the general region containing the correlation peak needs to be located. A correlator bank comprised of discrete points in the Doppler-shift/code-phase space expands about this starting estimate of Doppler shift, as well as the estimate for code phase, with as many offsets in each direction as a 3-σ uncertainty. The calculations performed by the correlator bank are comparable to a not-quite brute-force acquisition, and they provide several in-phase and quadrature accumulations, at each discrete point. The LMA nonlinear batch filter then acts as a multi-correlation vector discriminator of the observables carrier Doppler shift, code phase, carrier phase and carrier amplitude. It finds the best fit for the discrete correlations to an accumulation measurement model in order to yield the four observables that, along with this model, would deliver the peak correlation power of the accumulations. This is illustrated in Fig. 2, where the red dots are the correlator point measurements that the LMA interpolates to produce the observables at the theoretical peak. These observables are then fed into the measurement update portion of the extended Kalman filter/smooother that follows.

Rauch-Tung-Striebel Smoother

A nonlinear extended Kalman filter and the associated Rauch-Tung-Striebel smoother are used to create a smoothed sequence of carrier phase measurements over each 0.0228 segments. The nonlinear extended Kalman filter employs the measurement model associated with the outputs of the LMA, and a dynamics model for carrier phase, carrier Doppler shift, rate of change of carrier Doppler shift, code phase error, and carrier amplitude [2].

Fig. 2: Measured power of accumulations from a correlator bank, superimposed on theoretical power.

The dynamics model takes the linearized form:

\[ x_{k+1} = F_k x_k + \Gamma_k v_k \]  

where \( x_k = (\phi \quad \omega_D \quad \alpha \quad \Delta t_s \quad A)^T \) are the states of carrier phase, carrier Doppler shift, rate of change of carrier Doppler shift, code phase error, and carrier amplitude at the start of the \( k \)th accumulation interval \( \delta t_{DLL,k} = t_{DLL,k+1} - t_{DLL,k} \). \( v_k \) is the discrete-time Gaussian process noise vector associated with these states. The equations used to derive \( F_k \), the linearized dynamic model state matrix, and \( \Gamma_k \), the dynamic model process noise matrix, are given in Ref. [2]. The average carrier phase over the accumulation interval is

\[ \phi_{avg,k} = \left( 1 - \frac{1}{2} \delta t_{DLL,k} + \frac{1}{6} \delta t_{DLL,k}^2 \right) \left( \phi \quad \omega_D \quad \alpha \right)^T_k + (0 \quad 0 \quad 0 \quad 1) v_{\phi,k} \]  

where \( v_{\phi,k} \) is the Gaussian carrier phase process noise. Equation (2) is part of the Kalman filter’s linearized measurement model, which takes the form:

\[ y_k = H_{x,k} x_k + H_{w,k} v_k + w_k \]  

where the measurement vector \( y_k = (\omega_{D,avg,k} \Delta \text{smid}_k \phi_{avg,k} A_{IQ,k})^T \) consists of the average Doppler shift, code phase error, carrier
phase, and carrier amplitude, and refers to the optimal outputs of the LMA. The matrices \( H_{sk} \) and \( H_{vk} \) are again given in Ref. [2]. \( w_k \) is the discrete-time Gaussian measurement noise vector.

Data bits are unknown in this setting, so the absolute value of the phase innovation must be forced to be less than \( \pi/2 \). To do this, one starts out assuming data bit \( d_k = 1 \) at the beginning of the segment. Then one lets the hypotheses for the raw innovation be

\[
\begin{align*}
\nu_A \text{ raw} &= \phi_{av} - \bar{\phi}_{av} + (1 - d_k) \frac{\pi}{2} \\
\nu_B \text{ raw} &= \phi_{av} - \bar{\phi}_{av} + (1 + d_k) \frac{\pi}{2}
\end{align*}
\]

where \( \phi_{av} \) is the LMA carrier phase solution for the midpoint of the \( k \)th accumulation interval, and \( \bar{\phi}_{av} \) is the Kalman filter’s a priori expected phase determined from its measurement model. Hypothesis \( A \) implies that the data bit is +1, while hypothesis \( B \) implies that the data bit is -1. Then the actual innovation, assuming each hypothesis is correct, is constrained to be less than \( \pi \):

\[
\begin{align*}
\nu_A &= \nu_A \text{ raw} - 2\pi \text{round} \left( \frac{\nu_A \text{ raw}}{2\pi} \right) \\
\nu_B &= \nu_B \text{ raw} - 2\pi \text{round} \left( \frac{\nu_B \text{ raw}}{2\pi} \right)
\end{align*}
\]

Each innovation is then substituted for the phase part of the vector innovation \( v = y_k - H_{sk} x_k \) into the normalized equation that performs square-root information filtering dynamic propagation and measurement update [3]:

\[
\begin{pmatrix}
R_{vvk} & 0 \\
-H_{vk} F^{-1}_{k} \Gamma_{k} & R_{xxk} F^{-1}_{k} \Gamma_{k}
\end{pmatrix}
\begin{pmatrix}
\nu_k \\
x_{k+1} - \bar{x}_{k+1}
\end{pmatrix}
= \begin{pmatrix}
0 \\
0
\end{pmatrix}
\]

(6)

where \( R_{vvk} \) is the process noise square-root information matrix, \( R_{xxk} \) is the state square-root information matrix, \( F_{k} \) is, \( \Gamma_{k} \) is, and \( H_{vk} \) and \( H_{xx} \) are the measurement model matrices associated with process noise \( \nu_k \) and filter states \( x_{k+1} \), respectively. This equation is Q-R factorized to yield

\[
\begin{pmatrix}
R_{vvk} & R_{vxk+1} \\
0 & R_{xxk+1}
\end{pmatrix}
\begin{pmatrix}
x_k \\
\nu_k
\end{pmatrix}
= Q^T
\]

(7)

Finally, if \( |\nu_B| < |\nu_A| \) and \( \|\zeta_{A_{rk+1}}\|^2 - \|\zeta_{B_{rk+1}}\|^2 > 1 \), then the data bit is assumed to be -1, so for the next accumulation interval, use \( d_{k+1} = -d_k \). The last condition checks the sum of the measurement residuals, to see if they are significantly different in an ad-hoc way. At times when \( \nu_A \) and \( \nu_B \) are close in terms of multiples of \( \pi/2 \) but there has not been a bit flip, \( \|\zeta_{A_{rk+1}}\|^2 - \|\zeta_{B_{rk+1}}\|^2 \) will be close to zero. Similarly, if \( |\nu_B| > |\nu_A| \) and \( \|\zeta_{A_{rk+1}}\|^2 - \|\zeta_{B_{rk+1}}\|^2 > 1 \), then the data bit is assumed to be +1.

The a posteriori Kalman filter states are used as the centre of the next correlator bank, and the whole process is restarted for the next accumulation interval. This is repeated until the end of the 0.0228 s segment is reached, and constitutes the forward pass of the RTS smoother. As the data segment is being processed, the first two rows of Eq. (7) are retained in each accumulation interval. At the end of the segment, these two rows from all iterations, starting with the last, are fed into the backward pass of the smoother. The result is a smoothed time history of the carrier phase, carrier Doppler shift, rate of change of Doppler shift, code phase, and carrier amplitude for each antenna and each satellite. Note that these histories still contain the data gaps of the original signal.

III. DIFFERENTIAL CARRIER PHASE MEASUREMENT MODEL

It has been shown that a high-fidelity model for the single-differenced, total Doppler-shifted IF carrier phase between two antennas A and B for the same GPS satellite \( j \) is [4]

\[
\begin{align*}
\lambda \Delta \phi^{j}_{AB} &= (1 + a^{j}_{f_{1}}) \Delta \rho^{j}_{AB} \\
&+ c(1 + a^{j}_{f_{1}})(\Delta \delta_{TAB} + \Delta t_{mAB}) - c a^{j}_{f_{1}} \Delta t_{TAB} \\
&+ \lambda \Delta \gamma^{j}_{AB} + \lambda \Delta N^{j}_{AB} - \frac{40.3 \Delta \delta f_{C_{AB}}^{j}}{f^{2} \left(1 + a_{f_{1}}^{j}\right)} \\
&- \lambda (\Delta \delta \phi^{j}_{pwuAB} + \Delta \delta \phi^{j}_{mpAB} + \Delta N^{j}_{AB})
\end{align*}
\]

(8)

where \( \Delta(\cdot)_{AB} = (\cdot)_{A} - (\cdot)_{B} \) denotes the difference in the given quantity between antennas A and B. \( \lambda \) is the
nominal carrier wavelength, time parameter \( a^j_{t1} \) is part of satellite \( j \)'s navigation message, \( \rho^j_A \) is the range from receiver A to satellite \( j \), \( c \) is the speed of light \( \delta t_{RA} \) is receiver A's clock error, \( t_{ina} \) is the line bias, \( t_{RA} \) is the receiver time, \( \gamma^j_0 \) is the initial phase of receiver A's nominal carrier replica, \( \Delta N_{AB}^j \) is the carrier phase integer ambiguity between antennas A and B for satellite \( j \), \( f \) is the carrier frequency, \( \delta \phi^j_{pwu} \) is the phase wind up error, and the last two terms are the errors due to multipath and thermal noise.

Since the two receivers are linked in this case, the differential receiver clock error and differential receiver clock time drops out. The closeness of the antennas makes the difference in the ionospheric correction \( \Delta TEC_{AB}^j = 0 \). The antennas are aligned, so the differential phase wind-up term is also zero. Multipath and the \( a^j_{t1} \) parameter are usually negligible.

All the common-mode biases are lumped into a real-valued constant \( \delta \rho_0 \equiv c(1 + a^j_{t1}) \Delta t_{ina} + \lambda \Delta \gamma^j_0 + \Delta N_{AB}^j \), the last term of which is chosen arbitrarily for some satellite and time, since it cannot be separated observably from the other terms.

Decomposing \( \Delta \rho^j_{AB} = \hat{\rho}^j T A^T(q) b_{AB} \), where \( b_{AB} \) is the baseline vector pointing from antenna B to A in spacecraft coordinates, \( A \) is the inertial-to-spacecraft coordinate transform matrix dependent on the current quaternion \([5]\), and \( \hat{\rho}^j \) is the unit line-of-sight (LOS) vector pointing at satellite \( j \) from the rocket, the remaining equation is

\[
\lambda \Delta \phi^j_{AB} = \hat{\rho}^j T A^T(q) b_{AB} + \delta \rho_0 + \lambda \Delta N_{AB}^j + w^j \quad (9)
\]

The LOS vector is calculated from a secondary GPS receiver on the sub-payload. Equation (9) is the time-dependent measurement model with measurement white noise \( w^j \) whose covariance is the state covariance matrix element given by the RTS smoother. This model's geometries are illustrated in Fig. 3. The bias \( \delta \rho_0 \) is assumed to be constant over all data arcs, and each independent \( \delta N^j_p = \Delta N_{AB}^j(p) - \Delta N_{AB}^j(1) \) is assumed to be constant over each data arc \( p=1,...,P \). The baseline vector is fixed for all times.

![Fig. 3: Diagram of antennas, baseline vector, and LOS unit vector of spinning sub-payload.](image-url)
Fig. 4: Batch of single-differenced carrier phases from RTS smoother.

Fig. 5: Zoomed-in version of Fig. 4, showing one 0.0228 s segment of single-differenced carrier phase measurements.

IV. ATTITUDE BATCH FILTER

To apply the LMA for determining attitude from differential carrier phase, Eq. (9) has to be linearized around a current Levenberg-Marquardt solution guess. The process begins by stacking all the measurements into a measurement vector \( \mathbf{z} \equiv (\Delta \phi_{AB}(t_1) \ A \Delta \phi_{AB}(t_2) \ldots)^T \) for all satellites \( j=1,\ldots,J \) and all sample times. The nonlinear part of Eq. (9) is isolated as another stacked vector \( \mathbf{h} \equiv (\mathbf{q}^T A^T(q) b_{AB}(t_1) \ \mathbf{p}^T A^T(q) b_{AB}(t_2) \ldots)^T \). Then, the initial quaternion and spin-rate vector are expressed as the sums of current guesses plus perturbations:

\[
q_0 = \tilde{q}_0 + \Delta q_0 \quad (13a)
\]
\[
\omega_0 = \tilde{\omega}_0 + \Delta \omega_0 \quad (13b)
\]

\( \mathbf{h} \) is then expanded in a first-order Taylor series to yield a final normalized and linearized equation

\[
R_a[\mathbf{z} - h(\tilde{q}_0, \tilde{\omega}_0)]
= R_a(A_q \ A_\omega \ A_{\delta \rho_0} \ A_n)
\begin{pmatrix}
\Delta q_0 \\
\Delta \omega_0 \\
\delta \rho_0 \\
\delta N_1^2 \\
\vdots \\
\delta N_J^2
\end{pmatrix}
(14)
\]

where \( R_a \) is the measurement noise information matrix, \( A_q \) and \( A_\omega \) are the Jacobian matrices of the Taylor-series expansion evaluated at \( \tilde{q}_0 \) and \( \tilde{\omega}_0 \), and \( A_{\delta \rho_0} \) and \( A_n \) are coefficient matrices derived from the measurement model. To constrain the size of the perturbations and limit how far the linearized model deviates from the original, nonlinear one, two more rows are added to Eq. (14):

\[
\begin{pmatrix}
R_a[\mathbf{z} - h(\tilde{q}_0, \tilde{\omega}_0)] \ 0 \\
0 \\
\end{pmatrix}
= \begin{pmatrix}
R_a A_q Q_s & R_a A_\omega & R_a A_{\delta \rho_0} & R_a A_n \\
\sqrt{T} I_3 & 0 & 0 & 0 \\
0 & T \sqrt{T} I_3 & 0 & 0 \end{pmatrix}
\begin{pmatrix}
\Delta q_{0s} \\
\Delta \omega_0 \\
\delta \rho_0 \\
\delta N_1^2 \\
\vdots \\
\delta N_J^2
\end{pmatrix}
(15)
\]

These rows utilize a tuning parameter \( \gamma \) and nominal period of rotation \( T \) to enforce soft constraints on the total \( \Delta q_{0s} \) and \( \Delta \omega_0 \) perturbations in order to help the LMA guarantee convergence. In addition, the final linearized model of Eq. (15) uses the transformation \( \Delta q_0 = (\Delta q_{0s}) \otimes \tilde{q}_0 = Q_s(\tilde{q}_0) \Delta q_{0s} \) [6] to eliminate the 4x1 quaternion perturbation \( \Delta q_{0s} \) in favour of the 3x1 multiplicative quaternion perturbation \( \Delta q_{0s} \), where \( \otimes \) denotes quaternion multiplication. This substitution ensures that the perturbations are orthogonal to the nominal quaternion, which guarantees the unit-normalization of \( \tilde{q}_0 + \Delta q_{0s} \) to first-order in \( \Delta q_0 \).

The LMA is finally implemented as follows. First, an initial guess of the pair \( \tilde{q}_0 \) and \( \tilde{\omega}_0 \) is chosen by drawing \( \tilde{q}_0 \) from the space of all unit-normalized 4-
vectors, and letting \( \hat{\omega}_0 \) be the approximate, nominal angular velocity set by spin-stabilization. This guarantees global convergence. If the initial guesses of \( \tilde{q}_0 \) and \( \tilde{\omega}_0 \) are too far from their true values, the algorithm fails to converge. Thus, many initial guesses are tried. The algorithm begins by evaluating Equation (15) with the initial guesses of \( \tilde{q}_0 \) and \( \tilde{\omega}_0 \) and \( \gamma = 0 \) to produce the optimal least-squares perturbations \( \Delta q_{0s} \) and \( \Delta \omega_0 \). If the total norm of these perturbations is greater than 1, a new pair of initial guesses is chosen as before. This is an ad-hoc test that saves processing time, since large perturbations suggest the algorithm will not converge. If the total norm of the perturbations is less than 1, the algorithm continues as follows. The true nonlinear least-squares cost \( J \) is found by evaluating Eq. (14) with these initial guesses and suppressing the \( \Delta q_{0s} \) and \( \Delta \omega_0 \) perturbations to be zero. \( R_q(A_{\delta \rho_0}A_n) \) is Q-R factorized for a least-squares ambiguity decorrelation algorithm (LAMBDA), in order to determine the constant bias and integer ambiguities for this iteration [7]. Note that for every \( (\tilde{q}_0, \tilde{\omega}_0) \) there is an optimal set \( \{ \delta \rho_0, \delta N^x_1, \ldots, \delta N^x_k \} \) given by LAMBDA. The corresponding sum of the squared errors in Eq. (14), \( J \), is a measure of how well the pair \( \tilde{q}_0 \) and \( \tilde{\omega}_0 \) solves the problem.

The optimal perturbations \( \Delta q_{0s} \) and \( \Delta \omega_0 \) determined earlier by Eq. (15) are applied to the nominal quantities via Eq. (13), and the new quantities are put back into the zero-perturbation form of Eq. (14) to find the new nonlinear cost \( J_{\text{new}} \). If the \( J_{\text{new}} \) is less than \( J \), the \( \Delta q_{0s} \) and \( \Delta \omega_0 \) perturbations are applied permanently and the iteration process is continued until convergence. If not, the tuning parameter \( \gamma \) is increased to produce smaller optimal \( \Delta q_{0s} \) and \( \Delta \omega_0 \) perturbations. The algorithm usually passes its termination tests after less than ten iterations. Moreover, given that the rocket spin rate is 0.3 Hz, the half-second data gaps can be tolerated without aliasing.

V. ATTITUDE RESULTS

The following are the results of applying the attitude batch filter to the carrier phase measurements extracted from the raw GPS signal data received aboard MICA. These are graphed against the attitude results from the magnetometer and horizon crossing indicator on the same payload.

Figure 6 plots each of the quaternion components. These have been calculated for near the middle of the flight, and for 6 s batches. This time interval of the batches is chosen to encompass at least a couple of periods of rotation, but to be small compared to the time scale of coning angle variations. The \( q_0 \) solutions at the beginning of the batches, denoted by the x’s, are 4 s apart and are dynamically propagated forward in time, so there are 2 s of interpolation overlap between each batch. Their alignment with each other shows that the dynamic model is appropriate. These propagated quaternions are compared to those derived from the magnetometer/HCI. The \( q_0 \) batch solutions’ \( \pm 1-\sigma \) values are also plotted as red circles.

Figure 7 shows the discrepancies in the transverse axes between the attitude from carrier differential GPS and the attitude from the magnetometer/HCI. Figure 8 shows the corresponding discrepancies about the spin axis. There is a significant incongruity in the spin axis, as seen in Fig. 8. The source of this disagreement might be attributed to inconsistent coordinate frames, hardware timing errors, etc., but the issue is still being resolved. The total angle discrepancy is given in Fig. 9.

Angular velocity plots are presented in Fig. 10. Again, the \( \pm 1-\sigma \) values for the initial rate estimates are shown as red circles. Noting the small scale of the vertical axes, the discrepancies are small. The matching of the amplitude of oscillations in the
transverse axes suggests that the batch filter is returning the correct coning.

![Graph of angle discrepancy time history for transverse axes.](image)

**Fig. 7:** Angle discrepancy time history for transverse axes.

![Graph of angle discrepancy time history for spin axis.](image)

**Fig. 8:** Angle discrepancy time history for spin axis.

**VI. CONCLUSION**

This paper has presented a way to resolve 3-axis attitude and attitude-rate using only two GPS antennas, despite a highly dynamic environment and a low duty cycle in signal reception – one 0.0228 s of GPS data every 0.5 s of rocket flight. The 3-axis attitude and rate estimation algorithm solves a mixed real/integer problem, and eliminate the need to first remove the ambiguities at the possible cost of discarding useful information. The efficacy of this paper’s algorithm is demonstrated by application to a set of data from a sounding rocket experiment with substantial levels of coning motion. Despite its long signal gaps, the raw GPS data from this mission is successfully tracked by a using a bank of correlators, a Levenberg-Marquardt vector discriminator, and a Rauch-Tung-Striebel smoother to produce accurate differential carrier phases. These phases are processed in a batch filter that solves for the real-valued attitude solution and carrier phase integer ambiguities simultaneously. The results were reasonable when compared with those from a magnetometer and horizon crossing indicator, although there still looks to be some sort of time tagging or coordinate frame definition error.

![Graph of total angle discrepancy time history.](image)

**Fig. 9:** Total angle discrepancy time history.

**REFERENCES**


Fig. 10: Angular velocity component time histories.