Real-Time Multipath Estimation for Dual Frequency GPS Ionospheric Delay Measurements

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BIOGRAPHY

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ABSTRACT

An autoregressive moving average multipath estimator is developed using dual frequency GPS code and carrier-phase ionospheric delay measurements to approximate the multipath error contribution to this measurement and to excise it. This technique provides a means of improving the real-time multipath rejection of a GNSS receiver that operates from a fixed location to perform ionospheric remote sensing. The algorithm corrects the current differential code TEC measurement by aligning it with the best multipath estimate, which is calculated from an average of the previous days' satellite TEC measurements.

The effectiveness of this technique is demonstrated by comparing the filtered and unfiltered differential code TEC measurements over the same time period. The filter ran for two extended trials of four days at mid-latitude and twenty-five days at equatorial latitude. Dual frequency L1 C/A and L2C code data was collected at 1 Hz to obtain enough measurements for high resolution multipath estimates. After running the filter over four days, the standard deviation of TEC errors was reduced by approximately 8 TECU, and after twenty-five days, the standard deviation of TEC errors was reduced by 9.5 TECU.

I. INTRODUCTION

Multipath mitigation methods have been developed that exploit the repeatability of multipath errors between successive sidereal days for a fixed-location receiver [1]. This multipath repetition is the result of the nearly repeating elevation/azimuth of GPS satellites from day to day. The advantage of this system is that it strives to remove multipath effects from TEC measurements' based on differential L1/L2 PRN code phase measurements. This is easier than trying to remove multipath effects from code phase measurements on a single frequency, as in Ref. [2], because a double-differencing technique is not required to remove clock and tropospheric errors from the observables.

This goal is important because code-multipath-induced TEC errors limit the accuracy with which the bias can be removed from differential-carrier-phase-based TEC measurements [3]. This can be problematic for civilian dual-frequency TEC measurements that use the L1 C/A code and the L2C code. These codes' 1.023 MHz chipping rates admit large multipath errors that translate into differential code delay errors of 10s of TEC units.

The new method builds on an after-the-fact method of using differential code-based TEC to remove the bias from differential-carrier-phase-based TEC. One can record code and carrier TEC for a pass and subtract a bias from the carrier-phase based TEC time history to make the carrier-phase and code-phase TEC have equal means. This method suffers from two drawbacks: First, it cannot give absolute TEC in real-time at the accuracy levels of carrier-phase measurements. Second, its after-the-fact carrier-phase TEC measurements retain a residual bias error that is the residual average code multipath over the pass. Assuming no cycle slips, the new method enables real-time carrier-phase-level TEC accuracy, and it reduces the residual carrier-phase biases due to code multipath.

The present paper's method starts by differencing the TEC based on differential L1/L2 code delay and the TEC based on differential L1/L2 carrier phase advance. After removing the mean from this special double difference, the resulting observable consists primarily of code measurement errors due to differential L1/L2 multipath and receiver thermal noise. The observables from one day can be averaged with observables from preceding days. The algorithm matches measurements from a given day with those of the previous day by finding the day-to-day measurement time mapping that best aligns the satellite's current azimuth/elevation position with the closest position in the previous day's orbit. After the end of a given pass, the raw differential L1/L2 code-delay TEC measurements and the raw differential L1/L2 carrier-phase-advance TEC measurements are re-processed to produce a new code-based TEC multipath observable time history. This observable time history is then used to update the estimated code TEC multipath time history using an autoregressive moving average. Each independent azimuth/elevation of the given GPS satellite produces its own independent auto-regressive moving average code-based TEC multipath estimate. Thus, an entire estimated multipath time history is updated once per day.

Since multipath errors are periodic and repeat once per sidereal day, the multipath estimated from the previous day's update can be used to predict the current multipath error. The predicted differential-code TEC multipath for the current azimuth/elevation is used to correct the current real-time differential L1/L2 code-delay TEC measurement. The running average of these corrected code-based TEC measurements for the current pass is then used to remove the bias from the current real-time differential L1/L2 carrier-phase-advance TEC measurement by matching code and carrier running averages up to the current sample. The result is an accurate real-time TEC measurement.

After operating for a number of days, the effects of receiver thermal noise will be filtered out by the autoregressive moving average of the multipath history. The thermal noise term is generally considered to be a random variable that is uncorrelated from day to day, and thus after several days, the estimate will mostly represent the multipath error. To account for drift of the GPS orbit, the autoregressive moving average effectively assigns a weight to each current multipath error observable for a given azimuth/elevation and additional weights to each member of the set of nearest azimuth/elevation neighbors over the past days. The weights are largest for the most recent multipath observables and are smaller for older observables. The filtering time constant of the auto-regressive average can be tuned to average over a long window of days if the azimuth/elevation drifts very slowly. Alternatively, it can be tuned to a shorter window if the GPS satellite's orbit is drifting in a way that causes the azimuth/elevation tracks to build up significant variations over a number of days.

II. MEASUREMENTS

The dual frequency measurements used for this analysis were recorded using the Cornell/UT/ASTRA CASES GPS receiver [4] at two separate locations. The CASES receiver tracks GPS L1 C/A and L2C code on up to 12 dual-frequency channels in real-time with a Digital Signal Processor (DSP) and records the pseudorange and carrier-phase observables at a rate of once per second. The first set of data was collected at the Jicamarca Radio Observatory outside of Lima, Peru from 24 March to 18 April 2011. This receiver was used in conjunction with an Antcom 3G1215 antenna [5]. The second set of data was recorded in Ithaca, New York from 28 to 31 July 2011. This receiver was connected to a dual frequency L1/L2 Leica AR25 GNSS antenna [6] with a series of choke rings for rejecting multipath.

III. CODE DELAY TEC FILTER

To simulate real-time processing on the recorded data, only one time sample of differential code and carrier-phase TEC measurements was calculated during each iteration. The first day of data was processed without removing multipath estimates, in order to initialize the filter. On preceding days, the closest alignment of the current satellite position to the previous day's position was determined by solving for the offset, $\delta t(t)$, from a sidereal day, T_{SD}, that maximized the function

$$\hat{\rho}(t - T_{SD} + \delta t(t)) \cdot \hat{\rho}^{k+1}(t)^T \tag{1}$$

where $\hat{\rho}^{k+1}(t)$ is a unit-normalized 1x3 ECEF position vector of the satellite at the current time, t, of the $k+1^{st}$ day, and $\hat{\rho}(t - T_{SD} + \delta t(t))$ is a nx3 matrix of unit-normalized ECEF satellite positions from one sidereal day earlier. The time offset, $\delta t(t)$, is typically non-zero due to variations in a GPS satellite's orbit from day-to-day and has been observed to be as large as 86 seconds [7]. To save computational resources, Equation 1 was evaluated in the range of $-300 \leq \delta t(t) \leq 300$ seconds.

After obtaining the time offset, the multipath estimate $\Delta\widetilde{TEC}_{mp}$ from the previous day was removed from the differential code-delay-based TEC measurements:

$$\begin{aligned} TEC^{k+1}_{code\ corr}(t) &= TEC^{k+1}_{code}(t) \\ &\quad - \Delta\widetilde{TEC}^{k}_{mp}(t_{opt}) \end{aligned} \tag{2}$$

where t_{opt} is equal to $(t - T_{SD} + \delta t(t))$, i.e., the time of optimal position alignment with the previous day's position. The multipath estimate $\Delta\widetilde{TEC}_{mp}$ was calculated after a complete satellite pass. It represents a weighted sum of the multipath estimates from the previous and the current day.

$$\begin{aligned} \Delta\widetilde{TEC}^{k+1}_{mp}(t) &= \alpha \cdot \Delta\widetilde{TEC}^{k}_{mp}(t_{opt}) + (1-\alpha) \\ &\quad \cdot \left[TEC^{k+1}_{code}(t) - TEC^{k+1}_{cp\ corr}(t)\right] \end{aligned} \tag{3}$$

where $\alpha \in [0,1]$ is a parameter to weight the previous and current multipath estimates, and $TEC_{cp\ corr}(t)$ is the corrected differential carrier-phase, given by

$$\begin{aligned} TEC^{k+1}_{cp\ corr}(t) &= TEC^{k+1}_{cp}(t) \\ &\quad - \text{mean}\left[\Delta TEC^{k+1}(t)\right] \end{aligned} \tag{4}$$

$$\Delta TEC^{k+1}(t) = TEC^{k+1}_{code}(t) - TEC^{k+1}_{cp}(t) \tag{5}$$

The corrected differential carrier-phase removes most of the unknown bias from the differential carrier-phase-based TEC measurements by offsetting its values to the mean of the unambiguous differential code-delay-based TEC measurements. The difference $TEC^{k+1}_{code}(t) - TEC^{k+1}_{cp\ corr}(t)$ mainly consists of multipath errors and receiver thermal noise. After several days of filtering, the effects of the zero-mean receiver thermal noise average out, since its values are uncorrelated from day-to-day. Thus, the multipath estimate as the number of days of differential code TEC filtering increases.

The filter can be tuned by adjusting the values of α to favor either the current TEC measurements or the previous $\Delta\widetilde{TEC}_{mp}(t)$. Initially, values of α were set to be small ($\sim$ 0.5) to prevent variations from a single day from dominating the estimate, but as more days became integrated into the multipath estimate, α was increased. The α was never set above 0.8, however. This allowed the current TEC measurements have some influence on the next day's estimate.

IV. RESULTS

After running the filter for four consecutive days on all L1 C/A and L2C data, the multipath errors were reduced considerably. On average, the standard deviation of ΔTEC decreased by approximately 7.9 TECU for the PRNs in the data set by the fourth day.

The results for a single PRN are plotted in Figure 1. The first day of unfiltered data is shown in Figure 1a. The standard deviation of ΔTEC for the entire satellite pass on the first day was 16.7 TECU. The value of α was set to 0 to calculate $\Delta\widetilde{TEC}_{mp}(t)$, meaning that the multipath estimate for the second day only consisted of multipath estimates from the first day. On the second day, in Figure 1b, the corrected differential code delay values became less noisy and nearly all of the oscillations present in the first day of unfiltered data were eliminated. Typically, multipath errors were large, on the order of 10 to 30 TECU, and removing them substantially improved the TEC measurements. At the beginning and end of the satellite's pass, where the carrier-to-noise ratio is the lowest, a large amount of noise still remained after one day of filtering. The value $\alpha = 0.5$ was used to calculate the multipath estimate for day three, i.e, the estimate was composed of one half of the first day's multipath estimate and one half of the second day's multipath estimate. On the third day, in Figure 1c, the algorithm improves only slightly over the previous days. For some cases in this data set, the standard deviation of ΔTEC increased on the third day of filtering by less than a half of a TECU. The α parameter was increased to 0.6 after the third day's pass. This increase effectively means that we trust the cumulative estimate of multipath from the past sidereal days slightly more than the current multipath estimate based on the current day's differential TEC measurements. This estimate was then used to filter the fourth day of measurements, and the standard deviation improved to 8.2 TECU.

The filter was then run for 25 days with measurements recorded at the Jicamarca Radio Observatory. Instead of showing all 25 individual plots, the results are summarized in Figure 2, in which the standard deviation of ΔTEC of an entire satellite pass is plotted for each day. The red circles

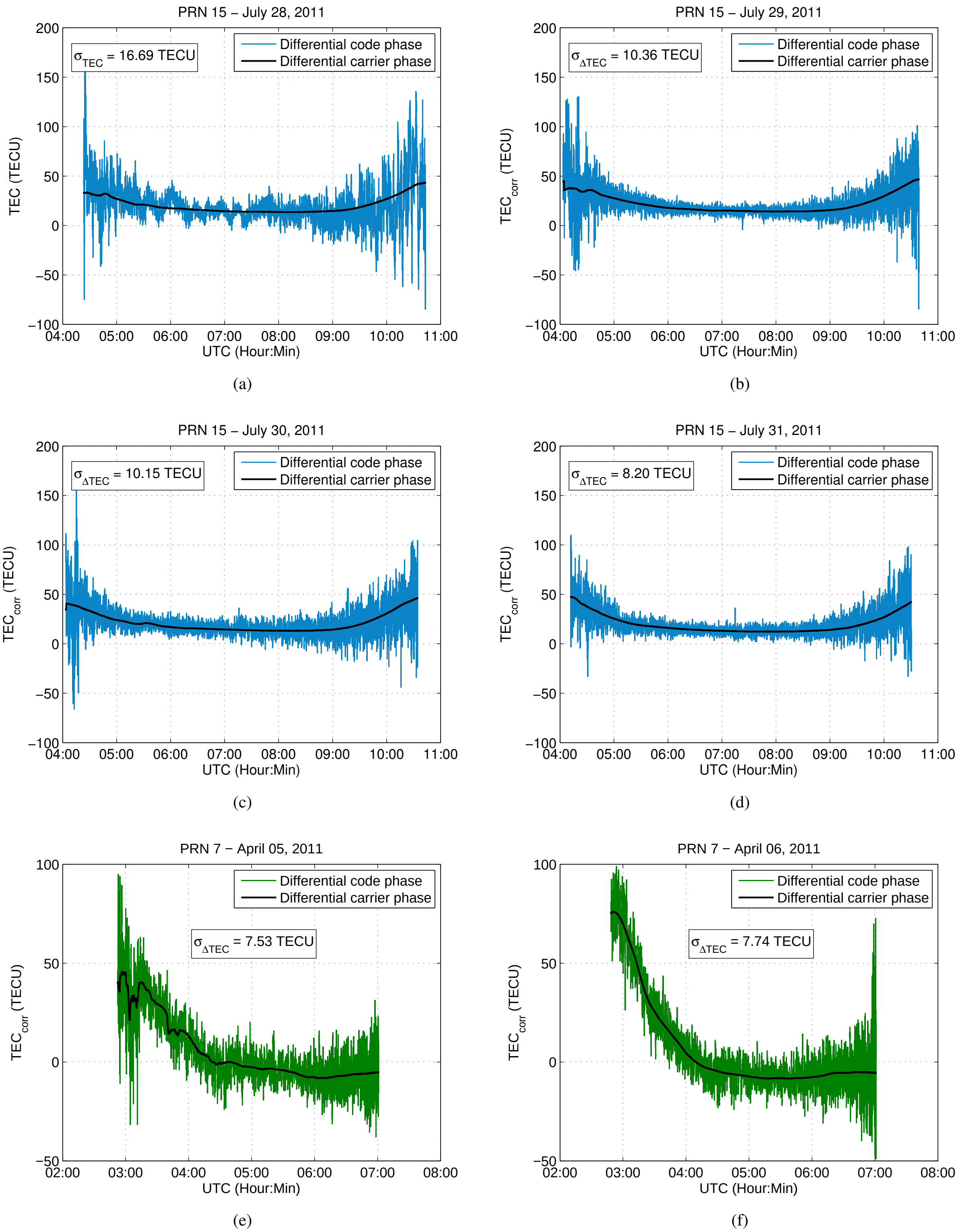

Figure 1: Differential code and carrier-phase TEC measurements from Ithaca, NY on 28-31 July 2011 for PRN 15 (a)-(d) and from Jicamarca on 5-6 April 2011 for PRN 7 (e)-(f). Panel (a) plots the unfiltered TEC measurements used to initialize the filter. Panels (b)-(d) show the filtered differential code TEC and the corrected carrier-phase TEC. The standard deviation of ΔTEC for the entire satellite pass is displayed above each plot.

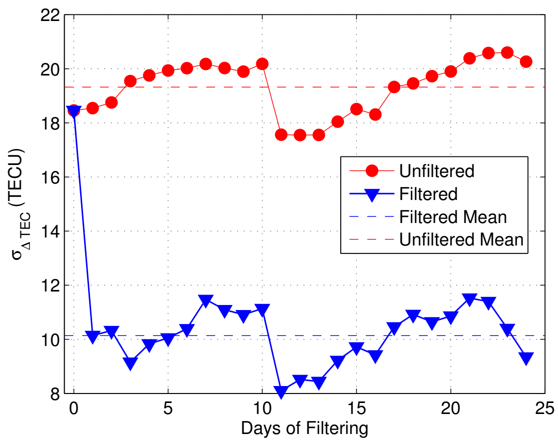

Figure 2: Standard deviation values of ΔTEC for PRN 7 over a 25 day run of the multipath filter. Each data point represents the standard deviation over the entire satellite pass. The filtered data is plotted with triangles, while the unfiltered standard deviation of ΔTEC is plotted with circles. The dashed horizontal lines at 19.18 and 10.14 TECU show the average $\sigma_{\Delta TEC}$ over days 1 to 24.

show the unfiltered data, while the blue triangles plot the filtered data. The multipath errors were reduced by about 8 TECU after only one day of filtering. In subsequent days, the filtered TEC sometimes improved or worsened from the first day, but in general, the standard deviations were well below the original unfiltered data. The variations in the standard deviations were most likely due to natural changes in the TEC from day to day. Since the data was recorded in an equatorial region, some fluctuations in $\sigma_{\Delta TEC}$ were expected and the filter adapted to these changes without significant degradation of the results.

Two days of filtered differential code TEC are shown in Figure 1e and 1f. These two plots demonstrate how the filter reacted to large variations in TEC that did not repeat daily. On the twelfth day of filtering, in Figure 1e, large changes in TEC occurred for approximately an hour starting at 3:00 UTC. The next day's results, however, were generally unaffected, as shown in Figure 1f. This is largely due to the fact that the filter coefficient α was increased after each sidereal day, effectively decreasing the variability of the multipath estimate from day to day.

To show how the filter improves the differential code delay measurements at different times throughout the satellite pass, the standard deviation of ΔTEC taken in half hour segments is plotted in Figure 3. For the July data set in Figure 3a, the largest improvements occur at the beginning and ends of the satellite pass. On the fourth day of filtering, $\sigma_{\Delta TEC}$ improves approximately 9 TECU in the first half hour segment and 16 TECU in the last half hour segment. The April data set in Figure 3b demonstrates similar trends when the satellite is rising and setting. Here, the standard deviation improves by about 12 TECU. Additionally, the filter removes errors associated with the large spike in $\sigma_{\Delta TEC}$ two hours into the satellite pass.

CONCLUSION

A real-time multipath filter for code-delay TEC measurements was presented and evaluated. The algorithm combined dual frequency code and carrier-phase measurements to form multipath estimates from the previous day, aligned the current satellite measurement with the closest location from the previous orbit, and then removed the multipath error associated with the optimal alignment time. The filter states were updated using an autoregressive moving average. The filter reduced the standard deviation of unwanted multipath errors by 8 to 10 TECU for a fixed receiver. Additionally, it was shown that the filter is relatively unaffected by non-sidereal repeating errors. These improvements are very important for improving the accuracy of stationary receivers monitoring space weather in real-time.

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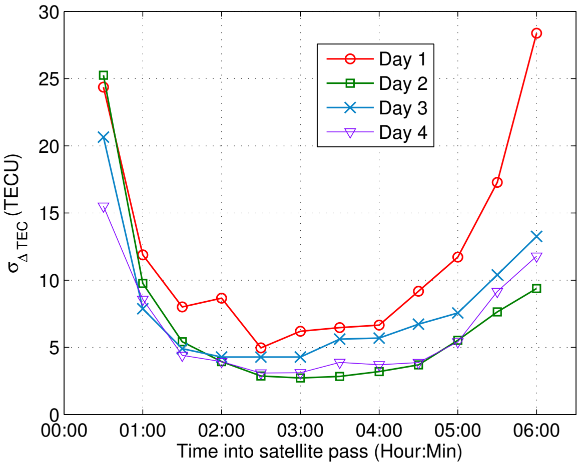
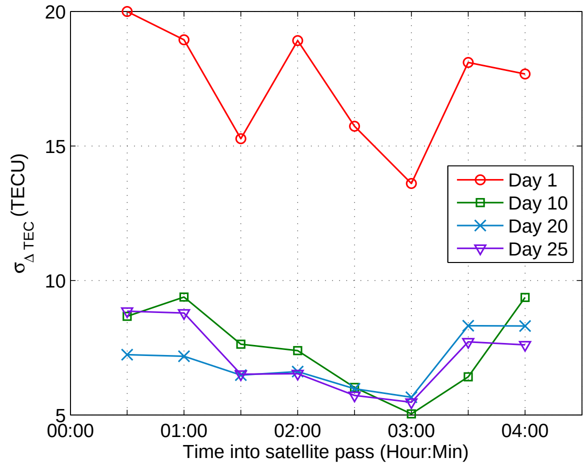


(a)



(b)

Figure 3: Standard deviation of ΔTEC taken in half hour segments throughout the satellite pass for each day. Panel (a) shows all four days from filtering PRN 15 in the July data set. Panel (b) shows the unfiltered first day, and the 10^{th}, 20^{th}, and 25^{th} days of filtered data from PRN 7 in the April data set.

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